RELATIVISTIC

SUPERINTENSE

LASER - ATOM

INTERACTIONS

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I. Space-translated Schrödinger equation

Standard Schrödinger equation for lab frame:

$$\left[\frac{1}{2m}\left(\mathbf{P} - \frac{e}{c}\mathbf{A}(\tau)\right)^2 + V(\mathbf{r})\right]\Psi = i\frac{\partial\Psi}{\partial t} \quad .$$

Laser pulse propagating in direction ***n***, with retardation:

$$\mathbf{A}(\tau) \quad , \quad \tau \equiv t - (\mathbf{r}\cdot\mathbf{n})/c \quad .$$

Dipole approximation: $\mathbf{A}(t)$.

Typical pulse form: $\mathbf{A}(t) = \mathbf{A_0}\, f(t) \sin \omega t$;

$f(t)$, pulse envelope (Gaussian, hypsec, etc.).

NR classical electron motion:

$$\gamma_{NR}(t) = -\frac{e}{mc}\int_{-\infty}^{t}\mathbf{A}(t')\,dt' \quad ;$$

initial conditions: $\gamma_{NR}(0) = (d\gamma_{NR} / dt)_0 = 0$.

Apply time-dependent space translation:

$$\mathbf{r} \qquad \mathbf{r} + \gamma_{NR}(t)$$

Defines "oscillating reference frame".
Classical electron at rest in the origin.

Space translated Schrödinger (STSE) equation:

$$\left[\frac{1}{2m}\mathbf{P}^2 + V(\mathbf{r} + \gamma_{NR}(t))\right]\Psi' = i\,\frac{\partial\Psi'}{\partial t}.$$

STSE, quite successful concerning physical insight & numerical integration.

For example: NR atomic stabilization in intense laser fields (1990).

At low intensity I, perturbation theory predicts growth of ionization with intensity. Intuitive.

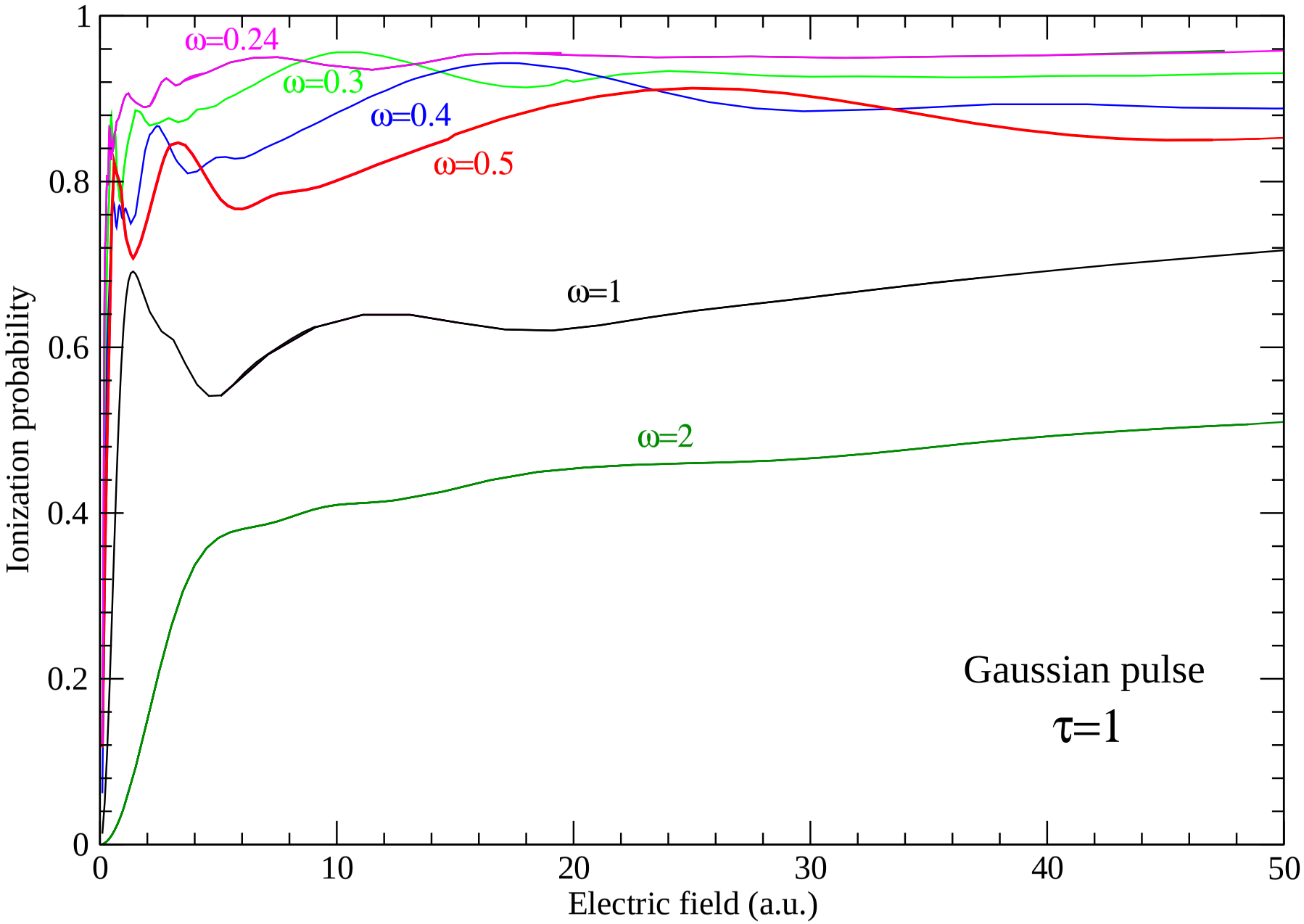
Pert.Th. breaks down at some I. Nonperturbative methods needed.

Surprise: ionization starts to decrease with I. Counter-intuitive.

Dynamic stabilization (DS), stabilization of ionization *probabilities*.

Fig.1 : P_{ion} for pulses with fixed Gaussian shape, when amplitude E_0 grows, at various frequencies ω; from Dondera et al (2002), and Stroe et al (in progress). DS at high ($\omega > 0.5$ au) & low ($\omega > 0.5$ au) freq.

Fig. 1 Dynamic Stabilization



II. Relativistic domain

What happens when ampl. E_0 is extremely large?

Relativity enters the scene via:

retardation
(variation of radiation phase within atom)

& relativistic dynamics
(law of motion changes from NR case).

The force on electron has now the Lorentz form:

$$\mathbf{F}_L = e\,\mathbf{E} + \frac{1}{c}\mathbf{v} \times \mathbf{H}$$

Classically: forward drift (in direction **n**).
The electron no longer returns to the origin at the end of the pulse.

Clasical relatativistic motion of an electron:

$$\gamma\ (t) = -\frac{e}{mc}\int_{-\infty}^{t-z/c}\mathbf{A}(\chi)\,d\chi$$

$$z(t) = \frac{e^2}{2m^2c^3}\int_{-\infty}^{t-z/c}\mathbf{A}^2(\chi)\,d\chi$$

Q.M. argument:

The center of the electron wave packet can be pushed far from the center of atom (Ehrenfest theorem); has finally no more overlap with the atom. Atomic survival & stabilization decrease.

ATI and HHG acquire new features.

Under relat. cond. Dirac eq should be used.

III. Dirac equation

Equation for lab frame (standard spin repres):

$$\left[c\alpha\left(\mathbf{P}-\frac{e}{c}\mathbf{A}(\tau)\right)+mc^2\beta+V(\mathbf{r})\right]\Psi = i\frac{\partial\Psi}{\partial t}$$

Ψ : 4 - comp "spinor" $(\Psi_1, \Psi_2, \Psi_3, \Psi_4)$
α, β : 4×4 anticomuting matrices

In a few cases can be solved analytically,

in many others numerically.

No numerical solution possible yet for case of

potential V(**r**) and intense **A**(τ) in 3D !

IV. GSTDE

Various approximate methods to treat the Dirac eq. (SFA etc.), or 1D models (numerically).

We have adopted a different approach, presented in the following.

We ≡ M. Boca, V. Florescu, M. Gavrila.

Is it possible to generalize the NR space translation operation to the relativistic case ?

Indeed, it is possible. But not as a simple space translation. Rather, as a transformation by a unitary operator: $T(t)$.

$T(t)$ defined as transforming the set of Volkov states into the set of free particle states (without the field).

Case of monochromatic, plane wave field considered by Krstic & Mittleman (PRA 1990).

We (BFG) have considered laser pulse case.

As a result, we finally obtain the generalized space-translated Dirac eq (GSTDE):

$$H'(t)\,\Psi' = i\frac{\partial \Psi'}{\partial t} \quad , \quad H'(t) \equiv \left(m^2c^4 + c^2\,\mathbf{P}^2\right)^{1/2}\beta + V' \quad ,$$

$$V' = T^{\dagger}\,V\,T$$

T and V' complicated integral operators.

GSTDE intractable in general case of an arbitrary relativistic situation.

Our goal: Apply Dirac equation to superintense laser-atom interactions (structure, ionization, laser-assisted collisions, etc).

Implies an initial cond. with :

low momenta p ($p/mc << 1$), small Z atoms ($\alpha Z << 1$), NR photons ($\omega << mc^2$).

Phenomena can become, nevertheless, relativistic during the interaction: the field forces the electron to oscillate with $v \leq c$ velocities in lab frame.

Our approach: study the motion with GSTDE, for superintense laser-atom interactions with NR initial conditions. Field can be arbitrarily intense.

We work within mathematical framework of:

"Low-Momentum Regime (LMR)".

Definition: Spinor $\Phi(\mathbf{r})$ is LMR if its Fourier expansion has a high momentum cut-off at ϖ , such that $\varpi/mc << 1$.

Question: Is it possible that a solution $\Psi'(t)$ of GSTDE, that is initially LMR, maintains this property at all t ?

(The laser pulse might modify its LMR character in time !)

We show that surprisingly the answer is affirmative !

As a consequence, GSTDE splits into two independent Pauli eqs. One describes wave packets with electronic initial conditions, the other w. p. with positronic initial cond. With spin ignored, these Pauli eqs. reduce to generalized Schrödinger eqs. Electronic case:

$$\left[\frac{1}{2m} \mathbf{P}^2 + V(\mathbf{r} + \gamma_e(\tau)) \right] \Psi' = i \frac{\partial \Psi'}{\partial t}.$$

General Conclusion:

Under LMR initial conditions (with spin ignored), GSTDE is equivalent to a generalized Schrödinger equation at all *t*.

Our eq. contains all the physical information contained in the Dirac eq. for current laser-atom interactions.

The eq. looks somewhat similar to the STSE. The difference resides in the form of the generalized potential.

As opposed to the Dirac eq., the BFG eq. is numerically tractable. A program has been developed by M. Boca & H.G. Muller (FOM Amsterdam) for its solution

BFG eq. is now being applied to physical processes.

Generalized Coulomb potential:

$$V'^{C} = -\frac{1}{|\mathbf{r} + \boldsymbol{\gamma}_e (t - (\mathbf{n} \cdot \mathbf{r})/c)|}$$

Has moving singularity $\mathbf{r}_S(t)$; this evolves along the space reflected trajectory of the classical electron. V'^{C} can undergo large time-dependent distortion.

Figs. 2 and 3 describe the case of :
$\omega = 0.05$ au; $E_0 = 10$ au; Gauss pulse, $\tau = 1$cy

Fig.2 shows the time-dependence of el. field E(t) of pulse.

Fig.3 shows the trajectory of the singularity of V'^{C} in the x (el. field), z(dir. of propag) plane. Also shown: level lines of V'^{C} in same plane, at some t, multiples of $T=2\pi/\omega$.

Fig.2

Electric field, Gauss pulse

$E_0 = 10$ au, $\omega = 0.05$ au

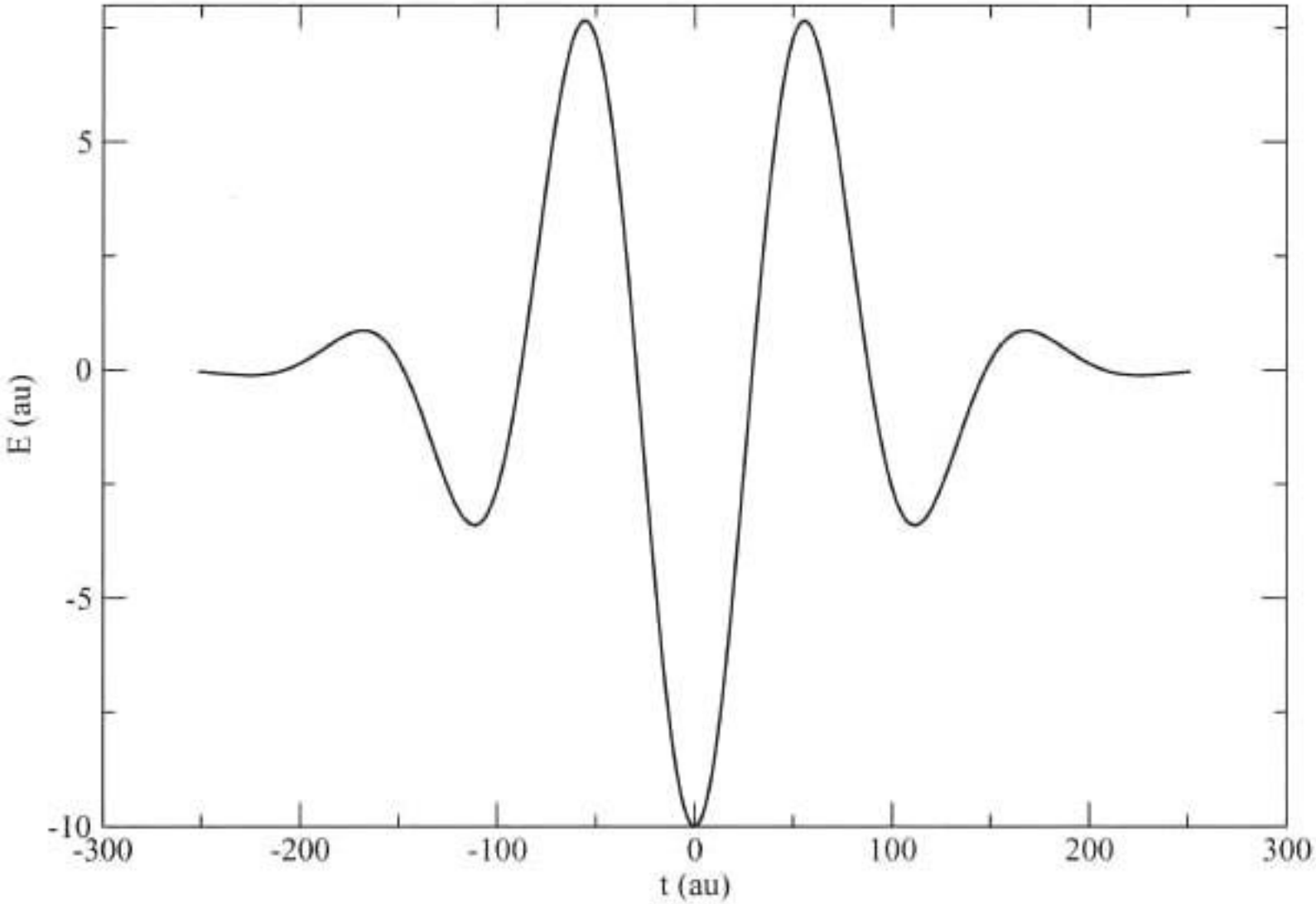


Fig.3 Generalized Coulomb Potential

Gauss Pulse, E_0= 10 au, ω=0.05 au